Teacher notes
Topic C

Standing waves

Suppose we look at standing waves on a string of length L with both ends fixed. The allowed wavelengths are

$$\lambda_n = \frac{2L}{n} \qquad n = 1,2,3,...$$

The corresponding frequencies are then

$$f_n = \frac{cn}{2L} \qquad n = 1,2,3,...$$

So the difference of 2 consecutive harmonics is $f_{n+1} - f_n = \frac{c(n+1)}{2L} - \frac{cn}{2L} = \frac{c}{2L}$ which happens to be the frequency of the first harmonic.

Example: Two consecutive harmonics on a string with both ends fixed have frequencies 240 Hz and 300 Hz. What is the frequency of the second harmonic?

The first harmonic has frequency 60 Hz so the second has frequency 120 Hz.

(You can answer this question by noting that all higher harmonics have frequencies which are multiples of the first harmonic frequency. Thus, we find the greatest common factor of 240 and 300. It is 60 Hz and so this is the first harmonic frequency. Why do we find the *greatest* common factor? 30 is also a factor. Choosing 30 Hz as the first harmonic frequency would make the 240 Hz wave the 8th harmonic and the 300 Hz wave the 10th harmonic. They two harmonics would then *not be consecutive* as the question requires.)

For a pipe of length L with one closed and one open end the corresponding result is:

$$\lambda_n = \frac{4L}{n} \qquad n = 1,3,5,... \text{ with frequencies } f_n = \frac{cn}{4L} \qquad n = 1,3,5,... .$$

The difference of 2 consecutive harmonics is $f_{n+2} - f_n = \frac{c(n+2)}{4L} - \frac{cn}{4L} = \frac{c}{2L}$ which happens to be double the frequency of the first harmonic.

Example: Two consecutive harmonics in a pipe with one open and one closed end have frequencies 400 Hz and 560 Hz. Which is the harmonic with frequency 400 Hz?

The difference in frequencies is 160 Hz and so the first harmonic has frequency 80 Hz. Then, $\frac{400}{80}=5$ so the 400 Hz harmonic is the fifth harmonic.

(You can answer this question by noting that all higher harmonics have frequencies which are odd multiples of the first harmonic frequency. Thus, we find the greatest odd common factor of 400 and 560. It is 80 Hz and so this is the first harmonic frequency.)